# Tennessee Math Standards

## Introduction

### The Process

The Tennessee State Math Standards were reviewed and developed by Tennessee teachers for Tennessee schools. The rigorous process used to arrive at the standards in this document began with a public review of the then-current standards. After receiving 130,000+ reviews and 20,000+ comments, a committee composed of Tennessee educators spanning elementary through higher education reviewed each standard. The committee scrutinized and debated each standard using public feedback and the collective expertise of the group. The committee kept some standards as written, changed or added imbedded examples, clarified the wording of some standards, moved some standards to different grades, and wrote new standards that needed to be included for coherence and rigor. From here the standards went before the appointed Standards Review Committee to make further recommendations before being presented to the Tennessee Board of Education for final adoption.

The result is Tennessee Math Standards for Tennessee Students by Tennesseans.

### Mathematically Prepared

Tennessee students have various mathematical needs that their K-12 education should address.

All students should be able to recall and use their math education when the need arises. That is, a student should know certain math facts and concepts such as the multiplication table, how to add, subtract, multiply, and divide basic numbers, how to work with simple fractions and percentages, etc. There is a level of procedural fluency that a student's K-12 math education should provide him or her along with conceptual understanding so that this can be recalled and used throughout his or her life. Students also need to be able to reason mathematically. This includes problem solving skills in work and non-work related settings and the ability to critically evaluate the reasoning of others.

A student's K-12 math education should also prepare him or her to be free to pursue post-secondary education opportunities. Students should be able to pursue whatever career choice, and its post-secondary education requirements, that they desire. To this end, the K-12 math standards lay the foundation that allows any student to continue further in college, technical school, or with any other post-secondary educational needs.

A college and career ready math class is one that addresses all of the needs listed above. The standards' role is to define what our students should know, understand, and be able to do mathematically so as to fulfill these needs. To that end, the standards address conceptual understanding, procedural fluency, and application.

### Conceptual Understanding, Procedural Fluency, and Application

In order for our students to be mathematically proficient, the standards focus on a balanced development of conceptual understanding, procedural fluency, and application. Through this balance, students gain understanding and critical thinking skills that are necessary to be truly college and career ready.

Conceptual understanding refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.

Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly. One cannot stop with memorization of facts and procedures alone. It is about recognizing when one strategy or procedure is more appropriate to apply than another. Students need opportunities to justify both informal strategies and commonly used procedures through distributed practice. Procedural fluency includes computational fluency with the four arithmetic operations. In the early grades, students are expected to develop fluency with whole numbers in addition, subtraction, multiplication, and division. Therefore, computational fluency expectations are addressed throughout the standards. Procedural fluency extends students' computational fluency and applies in all strands of mathematics. It builds from initial exploration and discussion of number concepts to using informal strategies and the properties of operations to develop general methods for solving problems (NCTM, 2014).

Application provides a valuable context for learning and the opportunity to practice skills in a relevant and a meaningful way. As early as Kindergarten, students are solving simple "word problems" with meaningful contexts. In fact, it is in solving word problems that students are building a repertoire of procedures for computation. They learn to select an efficient strategy and determine whether the solution(s) makes sense. Problem solving provides an important context in which students learn about numbers and other mathematical topics by reasoning and developing critical thinking skills (Adding It Up, 2001).

## Progressions

The standards for each grade are not written to be nor are they to be considered as an island in and of themselves. There is a flow, or progression, from one grade to the next, all the way through to the high school standards. There are four main progressions that are composed of mathematical domains/conceptual categories (see the Structure section below and color chart on the following page).

The progressions are grouped as follows:

| Grade | Domain/Conceptual Category |
|---|---|
| K | Counting and Cardinality |
| K-5 | Number and Operations in Base Ten |
| 3-5 | Number and Operations – Fractions |
| 6-7 | Ratios and Proportional Relationships |
| 6-8 | The Number System |
| 9-12 | Number and Quantity |
| K-5 | Operations and Algebraic Thinking |
| 6-8 | Expressions and Equations |
| 8 | Functions |
| 9-12 | Algebra and Functions |
| K-12 | Geometry |
| K-5 | Measurement and Data |
| 6-12 | Statistics and Probability |

State Standards – Mathematics Learning Progressions

<table>
<tr><th>Kindergarten</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS</th></tr>
<tr><td>Counting and Cardinality</td><td colspan="8"></td><td rowspan="3">Number and Quantity</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="2">Ratios and Proportional Relationships</td><td></td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations - Fractions</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td>Functions</td><td>Functions</td></tr>
<tr><td colspan="6">Geometry</td><td colspan="3">Geometry</td><td>Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td><td>Statistics and Probability</td></tr>
</table>

Each of the progressions begins in Kindergarten, with a constant movement toward the high school standards as a student advances through the grades. This is very important to guarantee a steady, age appropriate progression which allows the student and teacher alike to see the overall coherence of and connections among the mathematical topics. It also ensures that gaps are not created in the mathematical education of our students.

## Structure of the Standards

Most of the structure of the previous state standards has been maintained. This structure is logical and informative as well as easy to follow. An added benefit is that most Tennessee teachers are already familiar with it.

The structure includes:

- **Content Standards -** Statements of what a student should know, understand, and be able to do.

- **Clusters -** Groups of related standards. Cluster headings may be considered as the big idea(s) that the group of standards they represent are addressing. They are therefore useful as a quick summary of the progression of ideas that the standards in a domain are covering and can help teachers to determine the focus of the standards they are teaching.

- **Domains -** A large category of mathematics that the clusters and their respective content standards delineate and address. For example, *Number and Operations – Fractions* is a domain under which there are a number of clusters (the big ideas that will be addressed) along with their respective content standards, which give the specifics of what the student should know, understand, and be able to do when working with fractions.

- **Conceptual Categories –** The content standards, clusters, and domains in the $9^{th}$-$12^{th}$ grades are further organized under conceptual categories. These are very broad categories of mathematical thought and lend themselves to the organization of high school course work. For example, Algebra is a conceptual category in the high school standards under which are domains such as Seeing Structure in Expressions, Creating Equations, Arithmetic with Polynomials and Rational Expressions, etc.

## Standards and Curriculum

It should be noted that the standards are what students should know, understand, and be able to do; but, they do not dictate how a teacher is to teach them. In other words, the standards do not dictate curriculum. For example, students are to understand and be able to add, subtract, multiply, and divide fractions according to the standards. Although within the standards algorithms are mentioned and examples are given for clarification, how to approach these concepts and the order in which the standards are taught within a grade or course are all decisions determined by the local district, school, and teachers.

## Example from the Standards' Document for K – 8

Taken from 3rd Grade Standards:

| Measurement and Data (MD) | |
|---|---|
| **Cluster Headings** | **Content Standards** |
| **A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.** | 3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve contextual problems involving addition and subtraction of time intervals in minutes. *For example, students may use a number line to determine the difference between the start time and the end time of lunch.*<br><br>3.MD.A.2 Measure the mass of objects and liquid volume using standard units of grams (g), kilograms (kg), milliliters (ml), and liters (l). Estimate the mass of objects and liquid volume using benchmarks. *For example, a large paper clip is about one gram, so a box of about 100 large clips is about 100 grams. Therefore, ten boxes would be about 1 kilogram.* |

The domain is indicated at the top of the table of standards. The left column of the table contains the cluster headings. A light green coloring of the cluster heading (and codes of each of the standards within that cluster) indicates the major work of the grade. Supporting standards have no coloring. In this way, printing on a non-color printer, the standards belonging to the major work of the grade will be lightly shaded and stand distinct from the supporting standards. This color coding scheme will be followed throughout all standards K – 12. Next to the clusters are the content standards that indicate specifically what a student is to know, understand, and do with respect to that cluster. The numbering scheme for K-8 is intuitive and consistent throughout the grades. The numbering scheme for the high school standards will be somewhat different.

Example coding for grades K-8 standards:
**3.MD.A.1**
**3** is the grade level.
Measurement and Data (**MD**) is the domain.
**A** is the cluster (ordered by A, B, C, etc. for first cluster, second cluster, etc.).
**1** is the standard number (the standards are numbered consecutively throughout each domain regardless of cluster).

## Example from the Standards' Document for 9 – 12

Taken from Integrated Math 1 Standards:

**Algebra**

**Seeing Structure in Expressions (A.SSE)**

| Cluster Headings | Content Standards | Scope & Clarifications |
|---|---|---|
| **A. Interpret the structure of expressions.** | **M1.A.SSE.A.1** Interpret expressions that represent a quantity in terms of its context.* <br> **a.** Interpret parts of an expression, such as terms, factors, and coefficients. <br> **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity. | *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.* <br> *Tasks are limited to linear and exponential expressions, including related numerical expressions.* |

The high school standards follow a slightly different coding structure. They start with the course indicator (M1 – Integrated Math 1, A1 – Algebra 1, G – Geometry, etc.), then the conceptual category (in the example below – Algebra) and then the domain (just above the table of standards it represents – Seeing Structure in Expressions). There are various domains under each conceptual category. The table of standards contains the cluster headings (see explanation above), content standards, and the scope and clarifications column, which gives further clarification of the standard and the extent of its coverage in the course. A * with a standard indicates a modeling standard (see MP4 on p.11). The color coding is light green for the major work of the grade and no color for the supporting standards.

Example coding for grades 9-12 standards:
**M1.A.SSE.A.1**
Integrated Math 1 (**M1**) is the course.
Algebra (**A**) is the conceptual category.
Seeing Structure in Expressions (**SSE**) is the domain.
**A** is the cluster (ordered by A, B, C, etc. for first cluster, second cluster, etc.).
**1** is the standard number (the standards are numbered consecutively throughout each domain regardless of cluster).

## Tennessee State Math Standards

## The Standards for Mathematical Practice

Being successful in mathematics requires that development of approaches, practices, and habits of mind be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop within their students. These approaches, practices, and habits of mind can be summarized as “processes and proficiencies” that successful mathematicians have as a part of their work in mathematics.

Processes and proficiencies are two words that address the purpose and intent of the practice standards. Process is used to indicate a particular course of action intended to achieve a result, and this ties to the process standards from NCTM that pertain to problem solving, reasoning and proof, communication, representation, and connections. Proficiencies pertain to being skilled in the command of fundamentals derived from practice and familiarity. Mathematically, this addresses concepts such as adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive dispositions toward the work at hand. The practice standards are written to address the needs of the student with respect to being successful in mathematics.

These standards are most readily developed in the solving of high-level mathematical tasks. High-level tasks demand a greater level of cognitive effort to solve than routine practice problems do. Such tasks require one to make sense of the problem and work at solving it. Often a student must reason abstractly and quantitatively as he or she constructs an approach. The student must be able to argue his or her point as well as critique the reasoning of others with respect to the task. These tasks are rich enough to support various entry points for finding solutions. To develop the processes and proficiencies addressed in the practice standards, students must be engaged in rich, high-level mathematical tasks that support the approaches, practices, and habits of mind which are called for within these standards.

The following are the eight standards for mathematical practice:

| **Standards for Mathematical Practice** |
| --- |
| 1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>4. Model with mathematics.<br>5. Use appropriate tools strategically.<br>6. Attend to precision.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. |

A full description of each of these standards follows.

### *MP1: Make sense of problems and persevere in solving them.*

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### *MP2: Reason abstractly and quantitatively.*

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

### *MP3: Construct viable arguments and critique the reasoning of others.*

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

### *MP4: Model with mathematics.*

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

### *MP5: Use appropriate tools strategically.*

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a compass, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

### *MP6: Attend to precision.*

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.

### *MP7: Look for and make use of structure.*

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students see $7 \times 8$ equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

### *MP8: Look for and express regularity in repeated reasoning.*

Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Literacy Skills for Mathematical Proficiency

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others and analyze and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions.

| Literacy Skills for Mathematical Proficiency |
| --- |
| 1. Use multiple reading strategies. |
| 2. Understand and use correct mathematical vocabulary. |
| 3. Discuss and articulate mathematical ideas. |
| 4. Write mathematical arguments. |

### *Reading*

Reading in mathematics is different from reading literature. Mathematics contains expository text along with precise definitions, theorems, examples, graphs, tables, charts, diagrams, and exercises. Students are expected to recognize multiple representations of information, use mathematics in context, and draw conclusions from the information presented. In the early grades, non-readers and struggling readers benefit from the use of multiple representations and contexts to develop mathematical connections, processes, and procedures. As students' literacy skills progress, their skills in mathematics develop so that by high school, students are using multiple reading strategies, analyzing context-based problems to develop understanding and comprehension, interpreting and using multiple representations, and fully engaging with mathematics textbooks and other mathematics-based materials. These skills support Mathematical Practices 1 and 2.

### *Vocabulary*

Understanding and using mathematical vocabulary correctly is essential to mathematical proficiency. Mathematically proficient students use precise mathematical vocabulary to express ideas. In all grades, separating mathematical vocabulary from everyday use of words is important for developing an understanding of mathematical concepts. For example, a "table" in everyday use means a piece of furniture, while in mathematics, a "table" is a way of organizing and presenting information. Mathematically proficient students are able to parse a mathematical term, definition, or theorem, provide examples and counterexamples, and use precise mathematical vocabulary in reading, speaking, and writing arguments and explanations. These skills support Mathematical Practice 6.

### *Speaking and Listening*

Mathematically proficient students can listen critically, discuss, and articulate their mathematical ideas clearly to others. As students' mathematical abilities mature, they move from communicating through reiterating others' ideas to paraphrasing, summarizing, and drawing their own conclusions. A mathematically proficient student uses appropriate mathematics vocabulary in verbal discussions, listens to mathematical arguments, and dissects an argument to recognize flaws or determine validity. These skills support Mathematical Practice 3.

### *Writing*

Mathematically proficient students write mathematical arguments to support and refute conclusions and cite evidence for these conclusions. Throughout all grades, students write reflectively to compare and contrast problem-solving approaches, evaluate mathematical processes, and analyze their thinking and decision-making processes to improve their mathematical strategies. These skills support Mathematical Practices 2, 3, and 4.

# Mathematics | Grade 4

**The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout the 4th grade.**

**Operations and Algebraic Thinking**
Students build on their knowledge of multiplication and begin to interpret and represent multiplication as a comparison. They multiply and divide to solve contextual problems involving multiplicative situations, distinguishing their solutions from additive comparison situations. Students solve multi-step whole number contextual problems using the four operations representing the unknown as a variable within equations (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations). They apply appropriate methods to estimate and check for reasonableness. This is the first time students find and interpret remainders in context. Students find factors and multiples, and they identify prime and composite numbers. Students generate number or shape patterns following a given rule.

**Number and Operations in Base Ten**
Students generalize place value understanding to read and write numbers to 1,000,000, using standard form, word form, and expanded form. They compare the relative size of the numbers and round numbers to the nearest hundred thousand, which builds on 3rd grade rounding concepts. By the end of 4th grade, students should fluently add and subtract multi-digit whole numbers to 1,000,000. Students use strategies based on place value and the properties of operations to multiply a whole number up to four-digits by a one-digit number, and multiply two two-digit numbers. They use these strategies and the relationship between multiplication and division to find whole number quotients and remainders up to four-digit dividends and one-digit divisors (See Table 3 - Properties of Operations).

**Number and Operations-Fractions**
Students continue to develop an understanding of fraction equivalence by reasoning about the size of the fractions, using a benchmark fraction to compare the fractions, or finding a common denominator. Students extend previous understanding of unit fractions to compose and decompose fractions in different ways. They use the meaning of fractions and the meaning of multiplication as repeated addition to multiply a whole number by a fraction. Students solve contextual problems involving addition and subtraction of fractions with like denominators and multiplication of a whole number by a fraction (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations for whole number situations that can be applied to fractions). Students learn decimal notation for the first time to represent fractions with denominators of 10 and 100. They express these fractions and their equivalents as decimals and are able to read, write, compare, and locate these decimals on a number line.

**Measurement and Data**
Students know the relative sizes of measurement units within one system of units and are able to convert within the single system of measurement. They use the four operations to solve contextual problems involving measurement. Students build on their previous understanding of area and perimeter to generate and apply formulas for finding the area and perimeter of rectangles. Students also build on their understanding of line plots and solve problems involving fractions using operations appropriate for the grade. For the first time, students learn concepts of angle measurement.

**Geometry**
Students extend their previous understanding to analyze and classify shapes based on line and angle types. Students also use knowledge of line and angle types to identify right triangles. Students recognize and draw lines of symmetry for the first time.

## Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as "processes and proficiencies" that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

| Standards for Mathematical Practice |
|---|
| 1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>4. Model with mathematics.<br>5. Use appropriate tools strategically.<br>6. Attend to precision.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. |

## Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

| Literacy Skills for Mathematical Proficiency |
|---|
| 1. Use multiple reading strategies.<br>2. Understand and use correct mathematical vocabulary.<br>3. Discuss and articulate mathematical ideas.<br>4. Write mathematical arguments. |

## Operations and Algebraic Thinking (OA)

| Cluster Headings | Content Standards |
|---|---|
| **A. Use the four operations with whole numbers to solve problems.**<br>(See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations) | **4.OA.A.1** Interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations.<br><br>**4.OA.A.2** Multiply or divide to solve contextual problems involving multiplicative comparison, and distinguish multiplicative comparison from additive comparison. *For example, school A has 300 students and school B has 600 students: to say that school B has two times as many students is an example of multiplicative comparison; to say that school B has 300 more students is an example of additive comparison.*<br><br>**4.OA.A.3** Solve multi-step contextual problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. |
| **B. Gain familiarity with factors and multiples.** | **4.OA.B.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. |
| **C. Generate and analyze patterns.** | **4.OA.C.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* |

## Number and Operations in Base Ten (NBT)

| Cluster Headings | Content Standards |
|---|---|
| **A. Generalize place value understanding for multi-digit whole numbers.** | **4.NBT.A.1** Recognize that in a multi-digit whole number (less than or equal to 1,000,000), a digit in one place represents 10 times as much as it represents in the place to its right. *For example, recognize that 7 in 700 is 10 times bigger than the 7 in 70 because 700 ÷ 70 = 10 and 70 x 10 = 700.*<br><br>**4.NBT.A.2** Read and write multi-digit whole numbers (less than or equal to 1,000,000) using standard form, word form, and expanded form (e.g. the expanded form of 4256 is written as 4 x 1000 + 2 x 100 + 5 x 10 + 6 x 1). Compare two multi-digit numbers based on meanings of the digits in each place and use the symbols >, =, and < to show the relationship.<br><br>**4.NBT.A.3** Round multi-digit whole numbers to any place (up to and including the hundred-thousand place) using understanding of place value. |

| Cluster Headings | Content Standards |
| --- | --- |
| **B. Use place value understanding and properties of operations to perform multi-digit arithmetic.**<br>(See Table 3 - Properties of Operations) | **4.NBT.B.4** Fluently add and subtract within 1,000,000 using appropriate strategies and algorithms.<br><br>**4.NBT.B.5** Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.<br><br>**4.NBT.B.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. |

## Number and Operations - Fractions (NF)

**Limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.**

| Cluster Headings | Content Standards |
| --- | --- |
| **A. Extend understanding of fraction equivalence and comparison.** | **4.NF.A.1** Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{a \times n}{b \times n}$ or $\frac{a \div n}{b \div n}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. *For example,* $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$.<br><br>**4.NF.A.2** Compare two fractions with different numerators and different denominators by creating common denominators or common numerators or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Use the symbols >, =, or < to show the relationship and justify the conclusions. |

| Cluster Headings | Content Standards |
|---|---|
| **B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**<br><br>(See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations for whole number situations that can be applied for fractions.) | **4.NF.B.3** Understand a fraction $\frac{a}{b}$ with a > 1 as a sum of fractions $\frac{1}{b}$. *For example*, $\frac{4}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$.<br>**a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.<br>**b.** Decompose a fraction into a sum of fractions with the same denominator in more than one way (e.g., $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$), recording each decomposition by an equation. Justify decompositions by using a visual fraction model.<br>**c.** Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction.<br>**d.** Solve contextual problems involving addition and subtraction of fractions referring to the same whole and having like denominators |
| | **4.NF.B.4** Apply and extend previous understandings of multiplication as repeated addition to multiply a whole number by a fraction.<br>**a.** Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. *For example, use a visual fraction model to represent* $\frac{5}{4}$ *as the product* 5 × $\frac{1}{4}$, *recording the conclusion by the equation* $\frac{5}{4} = 5 \text{ x } \frac{1}{4}$.<br>**b.** Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ and use this understanding to multiply a whole number by a fraction. *For example, use a visual fraction model to express* 3 × $\frac{2}{5}$ *as* 6 × $\frac{1}{5}$, *recognizing this product as* $\frac{6}{5}$. *(In general,* $n \text{ x } \frac{a}{b} = \frac{(n \text{ x } a)}{b} = (n \text{ x } a) \text{ x } \frac{1}{b}$*)*<br>**c.** Solve contextual problems involving multiplication of a whole number by a fraction (e.g., by using visual fraction models and equations to represent the problem). *For example, if each person at a party will eat* $\frac{3}{8}$ *of a pound of roast beef, and there will be 4 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* |
| **C. Understand decimal notation for fractions and compare decimal fractions.** | **4.NF.C.5** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express,* $\frac{3}{10}$ *as* $\frac{30}{100}$ *and add* $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$.<br><br>**4.NF.C.6** Read and write decimal notation for fractions with denominators 10 or 100. Locate these decimals on a number line.<br><br>**4.NF.C.7** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Use the symbols >, =, or < to show the relationship and justify the conclusions. |

## Measurement and Data (MD)

| Cluster Headings | Content Standards |
|---|---|
| **A. Estimate and solve problems involving measurement.** | **4.MD.A.1** Measure and estimate to determine relative sizes of measurement units within a single system of measurement involving length, liquid volume, and mass/weight of objects using customary and metric units.<br><br>**4.MD.A.2** Solve one- or two-step real-world problems involving whole number measurements with all four operations within a single system of measurement including problems involving simple fractions.<br><br>**4.MD.A.3** Know and apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* |
| **B. Represent and interpret data.** | **4.MD.B.4** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.* |
| **C. Geometric measurement: understand concepts of angle and measure angles.** | **4.MD.C.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.<br>**a.** Understand that an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle.<br>**b.** Understand that an angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees and represents a fractional portion of the circle.<br><br>**4.MD.C.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.<br><br>**4.MD.C.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems *(e.g., by using an equation with a symbol for the unknown angle measure)*. |

## Geometry (G)

| Cluster Headings | Content Standards |
|---|---|
| **A. Draw and identify lines and angles and classify shapes by properties of their lines and angles.** | **4.G.A.1** Draw points, lines, line segments, rays, angles (right, acute, obtuse, straight, reflex), and perpendicular and parallel lines. Identify these in two-dimensional figures.<br><br>**4.G.A.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. Recognize right triangles as a category and identify right triangles.<br><br>**4.G.A.3** Recognize and draw lines of symmetry for two-dimensional figures. |

Major content of the grade is indicated by the light green shading of the cluster heading and standard's coding.

| | | | |
|---|---|---|---|
| | **Major Content** | | **Supporting Content** |

**Table 1 Common addition and subtraction situations**

| | Result Unknown | Change Unknown | Start Unknown |
|---|---|---|---|
| **Add to** | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?<br>$2 + 3 = ?$<br>(K) | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?<br>$2 + ? = 5$<br>(1st) | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?<br>$? + 3 = 5$<br>**One-Step Problem** (2nd) |
| **Take from** | Five apples were on the table. I ate two apples. How many apples are on the table now?<br>$5 - 2 = ?$<br>(K) | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?<br>$5 - ? = 3$<br>(1st) | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$<br>**One-Step Problem** (2nd) |
| | **Total Unknown** | **Addend Unknown** | **Both Addends Unknown**[2] |
| **Put Together/ Take Apart**[3] | Three red apples and two green apples are on the table. How many apples are on the table?<br>$3 + 2 = ?$<br>(K) | Five apples are on the table. Three are red and the rest are green. How many apples are green?<br>$3 + ? = 5$, $5 - 3 = ?$<br>(K) | Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?<br>$5 = 0 + 5$, $5 = 5 + 0$<br>$5 = 1 + 4$, $5 = 4 + 1$<br>$5 = 2 + 3$, $5 = 3 + 2$<br>(1st) |
| | **Difference Unknown** | **Bigger Unknown** | **Smaller Unknown** |
| **Compare**[4] | ("How many more?" version):<br>Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?<br>(1st) | (Version with "more"):<br>Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?<br>**One-Step Problem** (1st) | (Version with "more"):<br>Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have?<br>$5 - 3 = ?$ $? + 3 = 5$<br>**One-Step Problem** (2nd) |
| | ("How many fewer?" version):<br>Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie?<br>$2 + ? = 5$, $5 - 2 = ?$<br>(1st) | (Version with "fewer"):<br>Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have?<br>$2 + 3 = ?$, $3 + 2 = ?$<br>**One-Step Problem** (2nd) | (Version with "fewer"):<br>Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have?<br>**One-Step Problem** (1st) |

**K**: Problem types to be mastered by the end of the Kindergarten year.

**1st**: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.

**2nd**: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

**Table 2 Common multiplication and division situations**[1]

| | Unknown Product<br><br>$3 \times 6 = ?$ | Group Size Unknown<br>("How many in each group?" Division)<br>$3 \times ? = 18$, and $18 \div 3 = ?$ | Number of Groups Unknown<br>("How many groups?" Division)<br><br>$? \times 6 = 18$, and $18 \div 6 = ?$ |
|---|---|---|---|
| **Equal Groups** | There are 3 bags with 6 plums in each bag. How many plums are there in all?<br><br>*Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether? | If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?<br><br>*Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be? | If 18 plums are to be packed 6 to a bag, then how many bags are needed?<br><br>*Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have? |
| **Arrays,[2] Area[3]** | There are 3 rows of apples with 6 apples in each row. How many apples are there?<br><br>*Area example*. What is the area of a 3 cm by 6 cm rectangle? | If 18 apples are arranged into 3 equal rows, how many apples will be in each row?<br><br>*Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? | If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?<br><br>*Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? |
| **Compare** | A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?<br><br>*Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? | A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?<br><br>*Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? | A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?<br><br>*Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first? |
| **General** | $a \times b = ?$ | $a \times ? = p$, and $p \div a = ?$ | $? \times b = p$, and $p \div b = ?$ |

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

**Table 3 The properties of operations**

Here $a$, $b$ and $c$ stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

| | |
|---|---|
| *Associative property of addition* | $(a + b) + c = a + (b + c)$ |
| *Commutative property of addition* | $a + b = b + a$ |
| *Additive identity property of 0* | $a + 0 = 0 + a = a$ |
| *Associative property of multiplication* | $(a \times b) \times c = a \times (b \times c)$ |
| *Commutative property of multiplication* | $a \times b = b \times a$ |
| *Multiplicative identity property of 1* | $a \times 1 = 1 \times a = a$ |
| *Distributive property of multiplication over addition* | $a \times (b + c) = a \times b + a \times c$ |